NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2828

EFFECT OF A FINITE TRAILING-EDGE THICKNESS ON THE DRAG OF RECTANGULAR AND DELTA WINGS AT SUPERSONIC SPEEDS

By E. B. Klunker and Conrad Rennemann, Jr.

Langley Aeronautical Laboratory
Langley Field, Va.

Washington
November 1952

.K

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2828

EFFECT OF A FINITE TRAILING-EDGE THICKNESS ON THE DRAG OF RECTANGULAR AND DELTA WINGS AT SUPERSONIC SPEEDS

By E. B. Klunker and Conrad Rennemann, Jr.

SUMMARY

The effect of a finite trailing-edge thickness on the pressure drag of rectangular and delta wings with truncated diamond-shaped airfoil sections with a given thickness ratio is studied for supersonic Mach numbers, linearized theory being used to evaluate the surface pressures. In order to facilitate comparison with wings having sharp trailing edges, the position of maximum thickness and base height are determined for least pressure drag as functions of a base-pressure parameter. Comparison is then made between the drag of these wings and similar wings with a sharp trailing edge for various aspect ratios and thickness ratios as a function of stream Mach number. The calculations of the drag characteristics for these wings show that significant drag reductions are possible under some conditions at high supersonic speeds. These drag reductions are relatively independent of aspect ratio for the rectangular wings but depend considerably on aspect ratio for the delta wings; the smaller aspect ratios show the larger drag reductions. Calculations of the spanwise distribution of drag are included to compare further the effect of a base on the drag for different aspect ratios.

INTRODUCTION

A great deal of attention has been focused on the problem of determining supersonic profiles of minimum pressure drag. In reference 1, for example, profiles of minimum drag are determined by linearized theory for several auxiliary conditions, and in reference 2 an analysis is made for two structural conditions by using a nonlinear pressure relation. An interesting feature of these and similar investigations is that for certain conditions the profiles of minimum pressure drag have blunt trailing edges. The use of a finite trailing-edge thickness results in a reduction in the pressure drag of the forward surfaces of the airfoil (that is, the surface excluding the base) since the average absolute value of the slope of the airfoil surface is diminished. At high supersonic speeds, the reduction in pressure drag of the airfoil surface may exceed the additional base drag incurred for some conditions.

Consequently, an airfoil with a finite trailing-edge thickness may have less drag than a wing with a sharp trailing edge which satisfies the same structural conditions.

References 1 and 2 are concerned only with the effects of a finite trailing-edge thickness on two-dimensional airfoils. Of more practical interest is the effect of a base on wings of finite aspect ratio, particularly low-aspect-ratio wings, since it appears that these wings will be used at high supersonic flight speeds. An investigation of the drag of delta and rectangular wings is made in the present paper by using linearized theory to evaluate the surface pressures in order to determine the effect of a finite trailing-edge thickness on the drag of supersonic wings. In order to simplify the analysis, a truncated diamond-shaped airfoil - the shape for minimum drag for a given thickness ratio for two-dimensional airfoils - is considered. The location of maximum thickness and the trailing-edge thickness are determined for least drag in order to reduce the number of parameters involved and to indicate the drag reductions possible. These optimum wings, however, are not wings of minimum drag since the wing plan form and the form of the profile are specified and only its parameters are determined to give minimum drag. The drag coefficients for these wings are compared with those for similar wings with a sharp trailing edge for several aspect ratios and thickness ratios for the Mach number range from 2 to 8.

SYMBOLS

c	chord
c_r	root chord
t	thickness ratio
b	wing span
h	ratio of base height to chord or ratio of base area to wing area
$B = \frac{h}{t}$	
$1 - r$	position of maximum thickness, measured from leading edge, fraction of chord
Λ	sweep angle of leading edge
λ	wing slope in stream direction

λ_1	slope of front surface in plane $y = \text{Constant}$
λ_2	slope of rear surface in plane $y = \text{Constant}$
M	stream Mach number
μ	Mach angle, $\sin^{-1} \frac{1}{M}$
$\beta = \sqrt{M^2 - 1}$	
$n = \frac{\tan \Lambda}{\beta} = \frac{4}{\beta A}$	
U	stream velocity
w	vertical disturbance velocity component
ϕ	disturbance velocity potential
x,y,z ξ,η	Cartesian coordinates
S	wing area
S_b	base area
A	aspect ratio
P	pressure coefficient
P_b'	base pressure coefficient at any spanwise station
P_b	average base pressure coefficient
P_v	vacuum pressure coefficient
$-\frac{\beta P_b}{t}$	base-pressure parameter
C_D	wing pressure-drag coefficient
$\overline{C_D}$	pressure-drag coefficient of a wing with a sharp trailing edge
c_d	section pressure-drag coefficient

$\overline{c_d}$ section pressure-drag coefficient of a wing with a sharp trailing edge

Subscripts:

l wing of least drag

0.5 wing with maximum thickness location at midchord

ANALYSIS

The pressure drag of a wing is the drag due to the normal forces acting over its surfaces. The pressure drag of a wing with a finite trailing-edge thickness consists of a base drag and a pressure drag of the wing surface excluding the base. For supersonic flow, these two contributions to the pressure drag may be considered independently since disturbances from the trailing edge are not propagated upstream except to a small extent in the boundary layer. The normal pressures over the wing surface (excluding the base or trailing-edge surface) are determined mainly by the flow field outside the boundary layer where the effects of viscosity are negligible, whereas the base pressures are related directly to the viscous wake. In general, the base pressures are determined from experiment, whereas the surface pressures may be determined from inviscid-flow theory.

The surface pressures due to a thickness distribution may be calculated by linearized theory by using the concept of source distributions. Puckett (ref. 3) has shown that for a thin symmetric wing in a supersonic flow the required source strength at any point on the wing is proportional to the vertical component of the disturbance velocity (slope of the wing in the stream direction) at that point. Thus the determination of the disturbance velocity components, and consequently, the pressure, requires only an integration of the sources over the forward Mach cone.

The drag of a delta wing with a double-wedge airfoil section was calculated in reference 3. The drag of a delta wing with a truncated diamond-shaped airfoil section may be calculated by slightly modifying the equations of reference 3. Since the calculation of the drag of a rectangular wing is the same in principle as that of the delta wing, many of the details of the analysis are omitted. These may be found in reference 3 and elsewhere.

Rectangular wing.- For supersonic flow, the base drag may be considered separately from the remainder of the pressure drag. Hence, the pressure-drag coefficient C_D of a thin symmetric wing whose airfoil sections are all similar can be written as

$$C_D = \frac{2}{S}\int_S P\lambda \, ds - \frac{1}{S}\int_{S_b} P_b' \, ds$$

where P is the surface pressure coefficient, P_b' is the base pressure coefficient at any spanwise station, λ is the wing slope in the stream direction, S is the wing area, S_b is the base area, and ds is an element of area. If the average spanwise value of the base pressure coefficient is denoted by P_b, the wing pressure drag may be written as

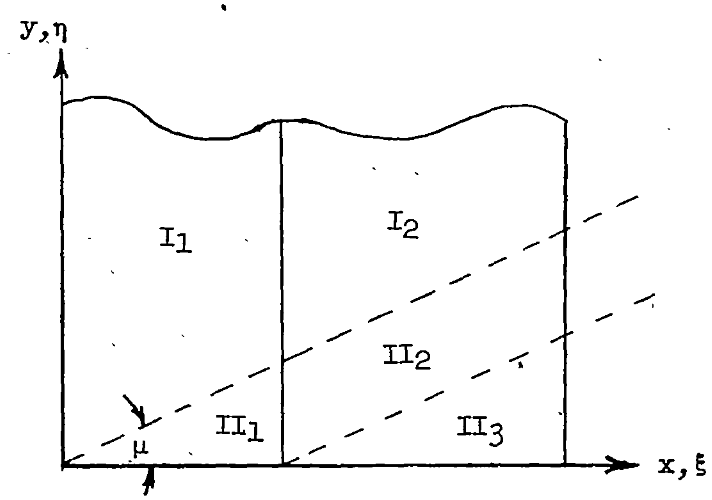
$$C_D = \frac{2}{S}\int_S P\lambda \, ds - P_b h \tag{1}$$

where

$$P_b = \int_0^1 P_b' \, d\left(\frac{y}{b/2}\right)$$

and the quantity h is the ratio of the base height to the chord, or, since all the airfoil sections for the wing are similar, it is the ratio of base area to wing area. The pressure drag of the wing surface excluding the base is considered in several parts for convenience.

The supersonic flow field past a rectangular wing is two-dimensional except in the region bounded by the wing tip and the tip Mach cone:



The surface pressure coefficient for the two-dimensional part of the flow field (region I, composed of the subregions I_1 and I_2 of the sketch) is

$$P = 2\frac{\lambda}{\beta}$$

where $\beta = \sqrt{M^2 - 1}$ and M is the stream Mach number. The drag coefficient of this two-dimensional-flow region then is

$$C_{D_I} = \frac{4}{\beta S}\int_{S_I}\lambda^2 ds$$

and for a double-wedge airfoil section with a finite trailing-edge thickness

$$C_{D_I} = \frac{4{\lambda_1}^2}{\beta}(1 - r)\left(1 - \frac{1 - r}{\beta A}\right) + \frac{4{\lambda_2}^2}{\beta}r\left(1 - \frac{2 - r}{\beta A}\right)$$

where $1 - r$ is the location of maximum thickness measured from the leading edge and A is the aspect ratio.

The wing plan forms and airfoil sections for both the rectangular and delta wings are shown in figure 1. From this figure the slopes of the forward and rear surfaces are found as

$$\lambda_1 = \frac{t}{2(1 - r)} \qquad \lambda_2 = -\frac{t}{2r}(1 - B)$$

where t is the thickness ratio and $B = \frac{h}{t}$. The drag coefficient for the two-dimensional region then becomes

$$C_{D_I} = \frac{t^2}{\beta}\left\{\frac{1 - B(1 - r)(2 - B)}{r(1 - r)} - \frac{1}{\beta A}\left[1 + \frac{(1 - B)^2(2 - r)}{r}\right]\right\} \tag{2}$$

for $\beta A > 2$. The condition $\beta A > 2$ requires that the tip Mach cones do not intersect on the wing surface.

The pressure coefficient at a point (x,y) in the tip region (composed of the subregions II_1, II_2, and II_3 of the sketch), which is bounded by the wing tip and the tip Mach cone, is found from the potential of the source distribution in the forward Mach cone. In reference 3 the required source strength at any point is shown to equal the vertical component of velocity w at that point. The velocity potential ϕ at $(x,y,0)$ in the tip Mach cone for a constant source strength is

$$\phi(x,y,0) = -\frac{w}{\pi}\int_0^y d\eta \int_0^{x-\beta(y-\eta)} \frac{d\xi}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2}} -$$

$$\frac{w}{\pi}\int_y^{\frac{x+\beta y}{\beta}} d\eta \int_0^{x+\beta(y-\eta)} \frac{d\xi}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2}}$$

and integrating with respect to ξ gives

$$\phi(x,y,0) = -\frac{w}{\pi}\int_0^{\frac{x+\beta y}{\beta}} \cosh^{-1}\frac{x}{\beta|y-\eta|}\, d\eta$$

The component of the disturbance velocity in the x-direction is found after some reduction as

$$\phi_x(x,y,0) = -\frac{w}{\beta\pi}\left(\pi - \cos^{-1}\frac{\beta y}{x}\right)$$

and from the linearized form of the Bernoulli equation the pressure coefficient is

$$P = -\frac{2\phi_x}{U} = \frac{2\lambda}{\beta\pi}\left(\pi - \cos^{-1}\frac{\beta y}{x}\right) \tag{3}$$

where U is the stream velocity and the airfoil slope is $\frac{w}{U}$.

The pressure coefficient for two-dimensional flow and the pressure coefficient given by equation (3) can be used to calculate the drag of the tip regions. The tip drag is determined by considering sources of strength $\lambda_1 U$ distributed over the whole tip area and sources of strength $(\lambda_2 - \lambda_1)U$ distributed over the regions II_2 and II_3. For the sources of strength $\lambda_1 U$ the pressure coefficient is given by equation (3) and the contribution to the drag for both wing tips is

$$C_{D_1}' = \frac{8\lambda_1^2}{\beta\pi S}\int_{S_{II_1}}\left(\pi - \cos^{-1}\frac{\beta y}{x}\right)ds + \frac{8\lambda_1\lambda_2}{\beta\pi S}\int_{S_{II_2}+S_{II_3}}\left(\pi - \cos^{-1}\frac{\beta y}{x}\right)ds$$

$$= \frac{t^2}{\beta^2 A}\,\frac{\pi - 1}{\pi} - \frac{t^2}{\beta^2 A}\,\frac{(1 - B)\left[1 - (1 - r)^2\right]}{r(1 - r)}\,\frac{\pi - 1}{\pi} \tag{4}$$

For sources of strength $(\lambda_2 - \lambda_1)U$, the pressure coefficient is the two-dimensional value in the region II_2 (since this region is outside the Mach cone emanating from the maximum-thickness line) and the pressure coefficient is given by equation (3), with x replaced by $x - c(1 - r)$, for the region II_3. This contribution to the drag of both tips is

$$C_{D_2} = \frac{8\lambda_2(\lambda_2 - \lambda_1)}{\beta S}\int_{S_{II_2}} ds + \frac{8\lambda_2(\lambda_2 - \lambda_1)}{\beta\pi S}\int_{S_{II_3}}\left(\pi - \cos^{-1}\frac{\beta y}{x - (1 - r)c}\right)ds$$

$$= \frac{2t^2(1 - B)\left[1 - B(1 - r)\right]}{\beta^2 Ar} + \frac{t^2(1 - B)\left[1 - B(1 - r)\right]}{\beta^2 A(1 - r)}\,\frac{\pi - 1}{\pi} \tag{5}$$

The total pressure drag of the rectangular wing then is

$$C_D = C_{D_I} + C_{D_1}' + C_{D_2} - P_b h$$

or, with $h = Bt$,

$$C_D = \overline{C_D} + \frac{t^2}{\beta}\left[\frac{B(B - 2)}{r} - \frac{B^2}{\pi\beta A} - \frac{\beta P_b}{t}B\right] \qquad (\beta A > 2) \tag{6}$$

where

$$\overline{C_D} = \frac{t^2}{\beta} \frac{1}{r(1 - r)}$$

is the drag of a rectangular wing with a sharp trailing edge. It is of interest to note that the drag of a rectangular wing with a finite trailing-edge thickness depends upon the aspect ratio; whereas the drag of a wing with a sharp trailing edge is independent of the aspect ratio.

Delta wing.- The pressure drag of a delta wing with a double-wedge airfoil section has been calculated in reference 3. The drag of a similar wing with a finite trailing-edge thickness can be found from the same relations provided the expression for the airfoil slope on the rear surface is suitably modified and the base drag is added. Since the effect of a finite trailing-edge thickness on the drag reduction is significant only at relatively high supersonic Mach numbers, only the condition of supersonic leading edges is presented. This corresponds to case 1 of reference 3.

The pressure drag of the delta wing C_{D_1}, excluding the base drag, is, from equations (41), (42), and (43) of reference 3,

$$C_{D_1} = \frac{8\lambda_1{}^2}{\beta\pi} G_1(n,r) + \frac{8\lambda_1\lambda_2}{\beta\pi}\left[\frac{\pi}{2} - G_1(n,r)\right] + \frac{8\lambda_2\left(\lambda_2 - \lambda_1\right)}{\beta\pi} r \frac{\pi}{2} \qquad (7)$$

where

$$G_1(n,r) = \frac{1 - r}{1 + r}\left[\frac{r}{\sqrt{1 - n^2}} \cos^{-1}n + \frac{1}{\sqrt{1 - n^2r^2}}\left(\frac{\pi}{2} + \sin^{-1}nr\right)\right]$$

and n is the ratio of the tangent of the leading edge sweep angle Λ to the cotangent of the Mach angle, that is, $n = \frac{\tan \Lambda}{\beta} = \frac{4}{\beta A}$. The pressure drag may be expressed as the sum of the drag contribution from equation (7) and the base drag. Hence

$$C_D = C_{D_1} - P_b h$$

where the average spanwise value of the base pressure coefficient P_b for the delta wing is

$$P_b = 2\int_0^1 P_b' \frac{c}{c_r} d\left(\frac{y}{b/2}\right)$$

where c is the local chord and c_r is the root chord. By substituting from equation (7) the drag coefficient becomes

$$C_D = \overline{C_D}\left[1 - B(1 - r)\right] - \frac{t^2}{\beta}\left[\frac{B(1 - B)}{r} + B\frac{\beta P_b}{t}\right] \tag{8}$$

where

$$\overline{C_D} = \frac{t^2}{\beta}\frac{2}{\pi(1 - r^2)}\left[\frac{\cos^{-1}n}{\sqrt{1 - n^2}} + \frac{1}{r\sqrt{1 - n^2r^2}}\left(\frac{\pi}{2} + \sin^{-1}nr\right)\right]$$

is the drag of a delta wing with a sharp trailing edge and $n < 1$.

<u>Least-drag wings</u>.- The equations for the drag of both the rectangular and delta wings each contain four parameters: r, B, $-\frac{\beta P_b}{t}$, and βA (or n). The effect of a finite trailing-edge thickness on the drag can be more clearly demonstrated by reducing the number of parameters. If the position of maximum thickness and the base height are determined to give the least drag for a given thickness ratio, then the quantities r and B may be expressed in terms of the parameters $-\frac{\beta P_b}{t}$ and βA (or n). The wings so determined, however, are not wings of minimum drag for a given thickness ratio in an exact sense since the wing plan form and the form of the profile are specified and only its parameters are determined to give minimum drag. The wings so determined are herein denoted as least-drag wings.

The values of r and B required to minimize the drag are found from the simultaneous solution of the equations

$$\frac{\partial C_D}{\partial r} = 0 \qquad \frac{\partial C_D}{\partial B} = 0 \tag{9}$$

in terms of the quantities $-\frac{\beta P_b}{t}$ and βA (or n). These values may then be substituted into equation (6) or (8) to yield the least drag coefficient for the particular wing plan form as a function of $-\frac{\beta P_b}{t}$ and βA (or n).

DISCUSSION

The wing profile geometry which minimizes the drag of a given wing plan form with plane surfaces is found as the simultaneous solution of equations (9). It is significant that the least-drag conditions for the rectangular wing are similar to those for a two-dimensional airfoil. The two-dimensional minimum-drag airfoil for a given thickness ratio, as determined by linearized theory, satisfies the condition $\lambda_1 = -\lambda_2$; further, the position of maximum thickness and the trailing-edge thickness depend only on the base-pressure parameter $-\frac{\beta P_b}{t}$ (see ref. 2, for example). For the rectangular wing the condition $\lambda_1 = -\lambda_2$ still holds for the least-drag condition (see appendix) however, the position of maximum thickness and the trailing-edge thickness depend both upon the base-pressure parameter and the aspect-ratio parameter βA. For the delta wing the two slopes are not equal and the geometric parameters for least drag are functions of $-\frac{\beta P_b}{t}$ and n.

The results of the solution of equations (9) are presented in figures 2(a) and 2(b) for the rectangular wing and the delta wing with supersonic leading edges. In these figures the position of maximum thickness $1 - r$ and the ratio of trailing-edge thickness to wing thickness B are given as functions of the parameters $-\frac{\beta P_b}{t}$ and βA (or n). These values may be substituted into equations (6) and (8) to determine the least drag coefficients. In figures 3(a) and 3(b) the ratio of the least drag coefficients C_{D_l} to the drag coefficient $\overline{C_{D_{0.5}}}$ (where $\overline{C_{D_{0.5}}}$ is the drag coefficient of a similar wing with a double-wedge airfoil section and maximum-thickness location at midchord) is presented as a function of $-\frac{\beta P_b}{t}$ for the rectangular and delta wings, respectively. Figures 2(a) and 3(a) show that for the rectangular wing both the geometric parameters and the drag coefficients differ but little (for moderate values of βA) from the values for the two-dimensional flow. Further, large pressure-drag reductions appear possible with the use of a finite trailing-edge thickness. Whether these large drag reductions can actually be realized, however, depends upon the base pressures experienced.

In order to assess the value of wings with a finite trailing-edge thickness, the elimination of the base pressure as a parameter is necessary. For this purpose, the base pressure coefficients presented in reference 2 have been utilized and are presented in figure 4 as a function of Mach number. Vacuum pressure coefficients are also presented for comparison. These values of the base pressure are representative for turbulent boundary layers.

The ratio of the drag coefficients $\frac{C_{D_l}}{\overline{C}_{D_{0.5}}}$ for the base pressures of figure 4 are presented in figures 5(a) and 5(b) for the rectangular and delta wings, respectively, as a function of the stream Mach number. As in the case of the two-dimensional airfoil, the percentage drag reduction possible with the use of a finite trailing-edge thickness is dependent to a large extent on the wing thickness ratio; the thicker wings show a substantially larger drag reduction at a given stream Mach number.[1] For example, the drag ratios for the rectangular wing at a Mach number of 6 are approximately 0.76 and 0.98 for thickness ratios of 0.10 and 0.06, respectively. For the rectangular wing, the effect of aspect ratio on the drag reduction is small compared with the effect of thickness ratio; for a given Mach number and thickness ratio the difference in $\frac{C_{D_l}}{\overline{C}_{D_{0.5}}}$ for aspect ratios of 1 and ∞ is generally less than 3 percent. In contrast, the effect of changes in thickness ratio and aspect ratio are of the same order of magnitude for the delta wing and the difference between $\frac{C_{D_l}}{\overline{C}_{D_{0.5}}}$ for aspect ratios of 1 and 4 may be more than 10 percent. The ratio $\frac{C_{D_l}}{\overline{C}_{D_{0.5}}}$ decreases with decreasing aspect ratio for both the rectangular and delta wings.

A clearer understanding of the effect of aspect ratio on the drag of the rectangular and delta wings is obtained by considering the spanwise distribution of section drag. The spanwise distribution of the quantity cc_d, where c is the section chord and c_d is the section drag coefficient, for these wings is presented in figure 6 for a stream Mach number of 5 and a thickness ratio of 0.10. For simplicity, the base pressure coefficient is taken as constant over the span.

---

[1]The drag coefficient of the two-dimensional airfoil of minimum drag $c_{d_{min}}$ is a linear function of t, whereas the drag coefficient of the airfoil of fixed geometry $\overline{c_d}$ increases as t^2 and the ratio $\frac{c_{d_{min}}}{\overline{c_d}}$ decreases with increasing values of the thickness ratio.

The pressures both inside and outside the Mach cone are unaffected by a change in aspect ratio for the rectangular wing with a specified airfoil section. Thus, changing the aspect ratio only changes the proportion of the two-dimensional region to the tip region and the spanwise distribution of drag changes similarly. Since the airfoil geometry for least drag depends slightly upon the aspect ratio, the pressures, and, consequently, the distribution of drag for the wing of least drag vary somewhat with aspect ratio. Figure 6 shows that the drag distributions in the tip regions are similar for different aspect ratios. That is, the basic distribution of drag within the tip does not change as the aspect ratio is changed although the proportion of tip area to wing area changes with a change in aspect ratio. Moreover, in a large part of the tip region, the difference between $\overline{c}_{d_{0.5}}$ and c_d is greater than in the two-dimensional region. Consequently, the ratio of $\frac{C_{D_l}}{\overline{C}_{D_{0.5}}}$ decreases with decreasing aspect ratio since the tip region is a larger percentage of the total wing area for the lower aspect ratios.

A change in aspect ratio for the delta wing necessarily alters the leading-edge sweep angle, and the pressures both inside and outside the Mach cone from the apex are changed. As shown in figure 6, a change in aspect ratio also alters the form of the spanwise distribution of drag for the airfoil of fixed geometry. For the delta wing of least drag, however, the distributions for the different aspect ratios tend to be similar; the section drag approaches a linear variation across the span.

CONCLUDING REMARKS

The calculations of the drag characteristics for rectangular and delta wings for a given thickness ratio show that significant drag reductions are possible under certain conditions at high supersonic speeds with the use of a finite trailing-edge thickness. The Mach number at which a blunt-trailing-edge wing has less drag than a similar wing with a sharp trailing edge is dependent upon the wing plan form, aspect ratio, and thickness ratio. For a given Mach number and aspect ratio, the thicker wings show a substantially larger drag reduction through the use of a finite trailing-edge thickness. The effect of aspect ratio on the drag reduction for the rectangular wings is small, whereas it is of the same order of magnitude as the effect of thickness for the delta wing. The ratio of the least drag coefficient of a wing with a blunt base to the drag coefficient of a similar wing with a sharp trailing edge decreases with decreasing aspect ratio. For the rectangular wing the spanwise distributions of drag in the tip region are similar for all aspect ratios, although the proportion of tip area to wing area changes with changing aspect ratio. For the delta wing with a fixed airfoil section, the form of the spanwise drag distribution is altered as the aspect ratio is changed, whereas for the least-drag delta wings the distributions for different aspect ratios tend to be similar. Although the analysis of this paper is confined to rectangular and delta wings with a given thickness ratio, the results suggest that significant drag reductions may be possible by the use of a finite trailing-edge thickness with other plan forms. It may be expected that, similar to the result for the two-dimensional airfoil (NACA TN 2264), the relative drag reduction possible by the use of a finite trailing-edge thickness with other structural conditions (given area, torsional stiffness, and so forth) is greater than for a given thickness ratio.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., September 15, 1952.

APPENDIX

LEAST-DRAG CONDITIONS FOR RECTANGULAR WING

The airfoil of minimum drag for a given thickness ratio in a two-dimensional supersonic flow is a truncated double wedge. Within the scope of linearized theory, λ_1 is then equal to $-\lambda_2$; that is, the slopes over the front and rear surfaces are equal (see ref. 2, for example). It is interesting to note that this same condition also applies to the rectangular wing of least drag with truncated diamond-shaped airfoil sections.

The drag of a rectangular wing with a truncated diamond-shaped airfoil section is (eq. (6))

$$C_D = \overline{C_D} + \frac{t^2}{\beta}\left[\frac{B(B - 2)}{r} - \frac{B^2}{\pi\beta A} - \frac{\beta P_b}{t} B\right]$$

where

$$\overline{C_D} = \frac{t^2}{\beta} \frac{1}{r(1 - r)}$$

The values of r and B required to give least drag are found from the simultaneous solution of the equations

$$\frac{\partial C_D}{\partial r} = 0 \qquad \frac{\partial C_D}{\partial B} = 0 \tag{A1}$$

From the first of equations (A1)

$$B = \frac{\frac{2}{r} - \left(-\frac{\beta P_b}{t}\right)}{\frac{2}{r} - \frac{2}{\pi\beta A}} \tag{A2}$$

and from the second of equations (A1)

$$B = \frac{1 - 2r}{1 - r} \tag{A3}$$

After eliminating B from equations (A2) and (A3), r is found as

$$r = \frac{2 + \frac{2}{\pi\beta A} - \left(-\frac{\beta P_b}{t}\right)}{\frac{4}{\pi\beta A} - \left(-\frac{\beta P_b}{t}\right)} \tag{A4}$$

The slopes of the front and rear surfaces are $\lambda_1 = \frac{t}{2(1 - r)}$ and $\lambda_2 = -\frac{t}{2r}\left(1 - B\right)$, respectively. Substituting equation (A3) for B in the equation for λ_2 results in the relation

$$\lambda_2 = -\frac{t}{2r}\left(1 - \frac{1 - 2r}{1 - r}\right) = -\frac{t}{2(1 - r)} = -\lambda_1$$

Consequently, the condition for least drag of the rectangular wing with a truncated diamond-shaped airfoil section is $\lambda_1 = -\lambda_2$.

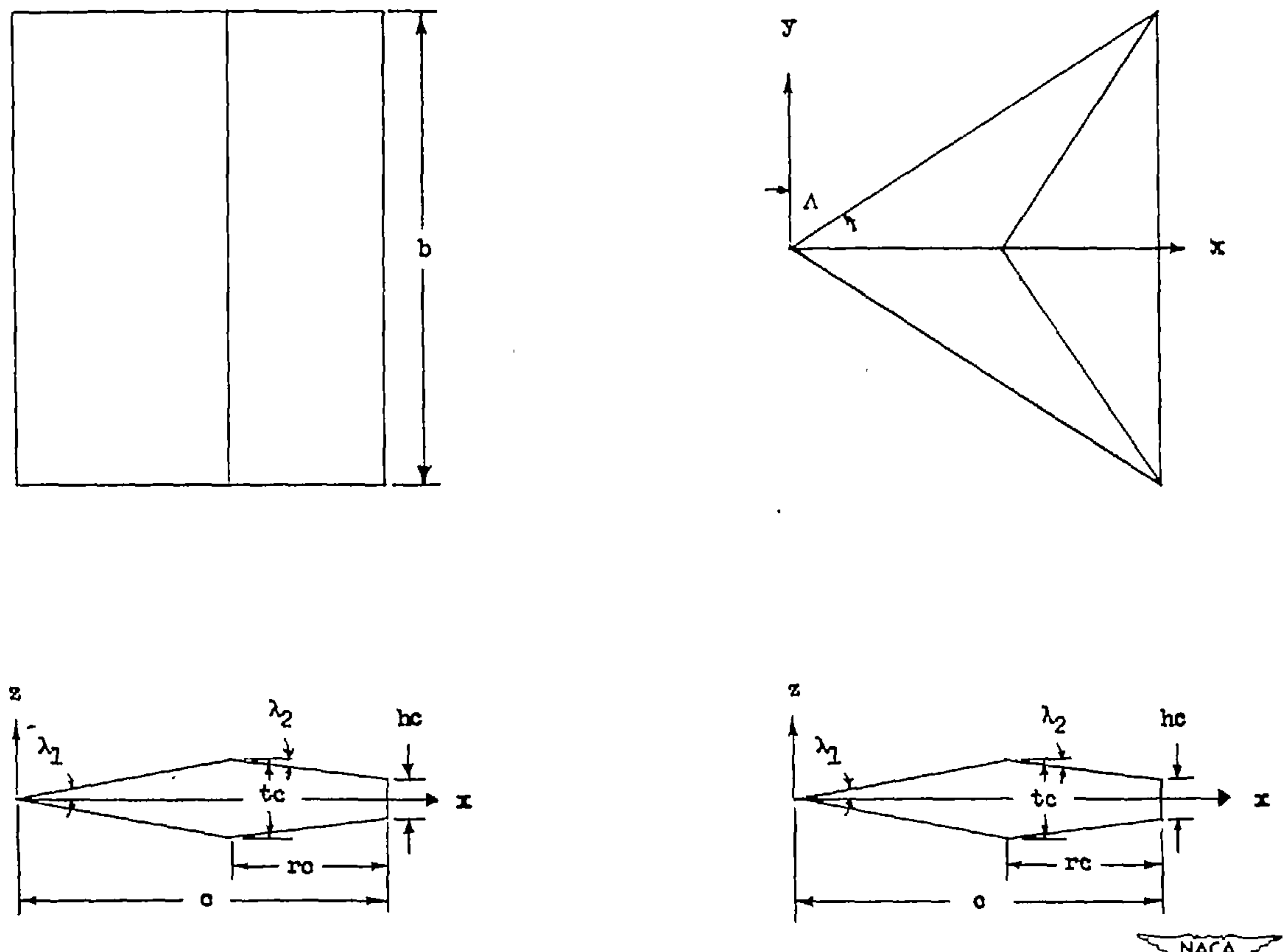
From equations (A2) and (A4) it is seen that r and B depend on both the base pressure parameter $-\frac{\beta P_b}{t}$ and the aspect ratio parameter βA. This is in contrast with the two-dimensional case where r and B depend only on $-\frac{\beta P_b}{t}$.

REFERENCES

1. Chapman, Dean R.: Airfoil Profiles for Minimum Pressure Drag at Supersonic Velocities - General Analysis With Application to Linearized Supersonic Flow. NACA TN 2264, 1951.

2. Klunker, E. B., and Harder, Keith C.: Comparison of Supersonic Minimum-Drag Airfoils Determined by Linear and Nonlinear Theory. NACA TN 2623, 1952.

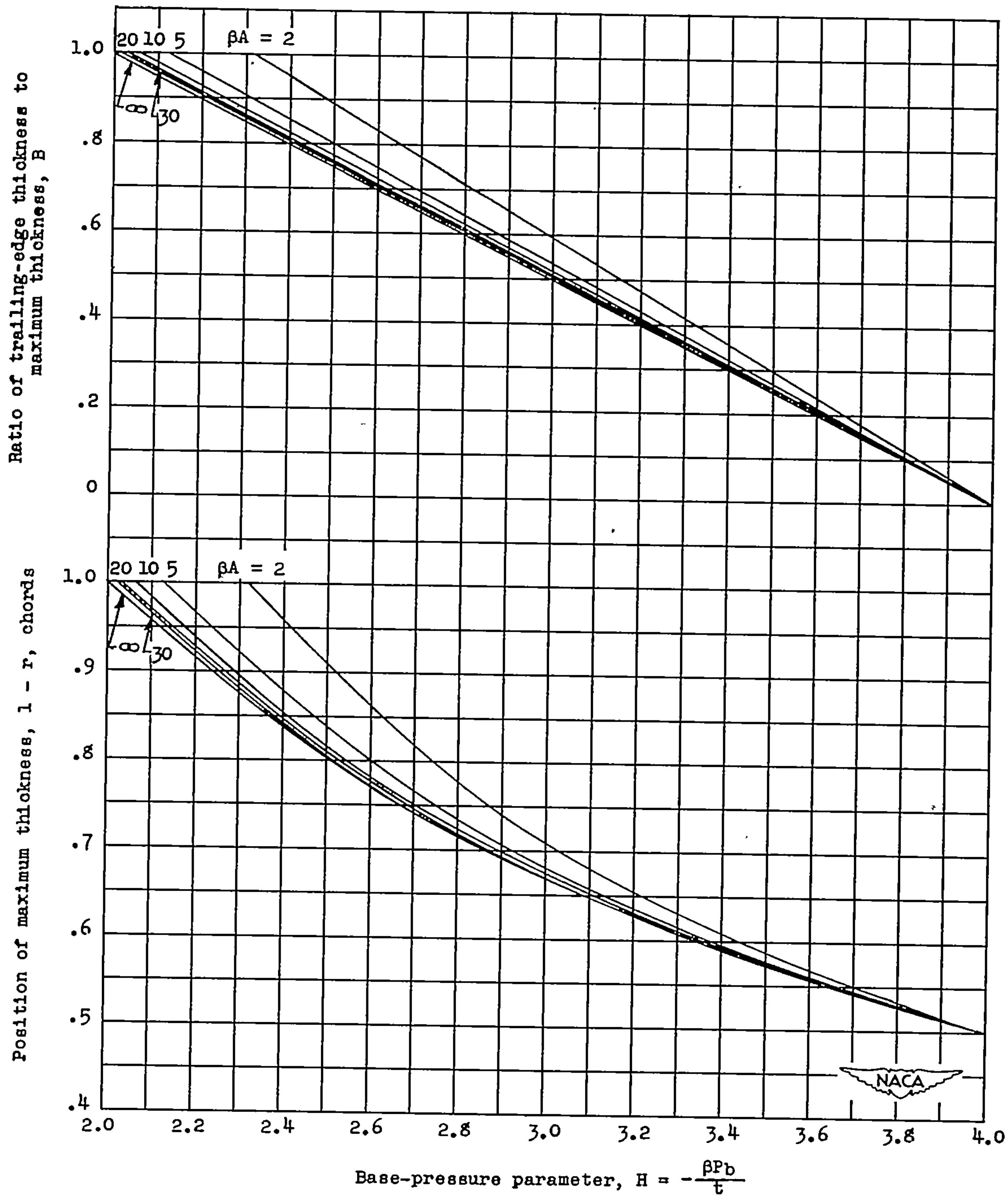
3. Puckett, Allen E.: Supersonic Wave Drag of Thin Airfoils. Jour. Aero. Sci., vol. 13, no. 9, Sept. 1946, pp. 475-484.



Rectangular wing

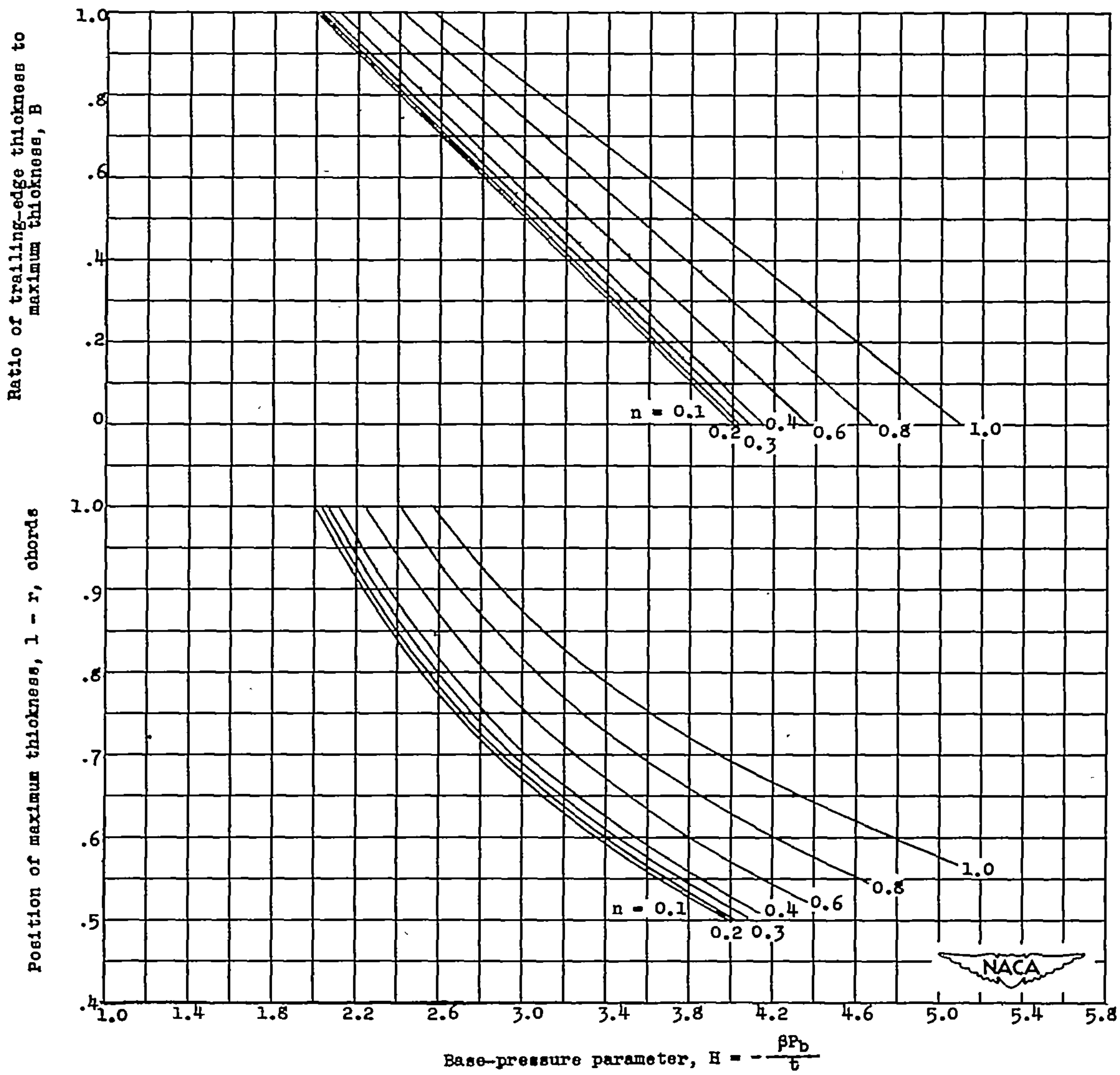
Delta wing

Figure 1.- Wing geometry for rectangular and delta wings.



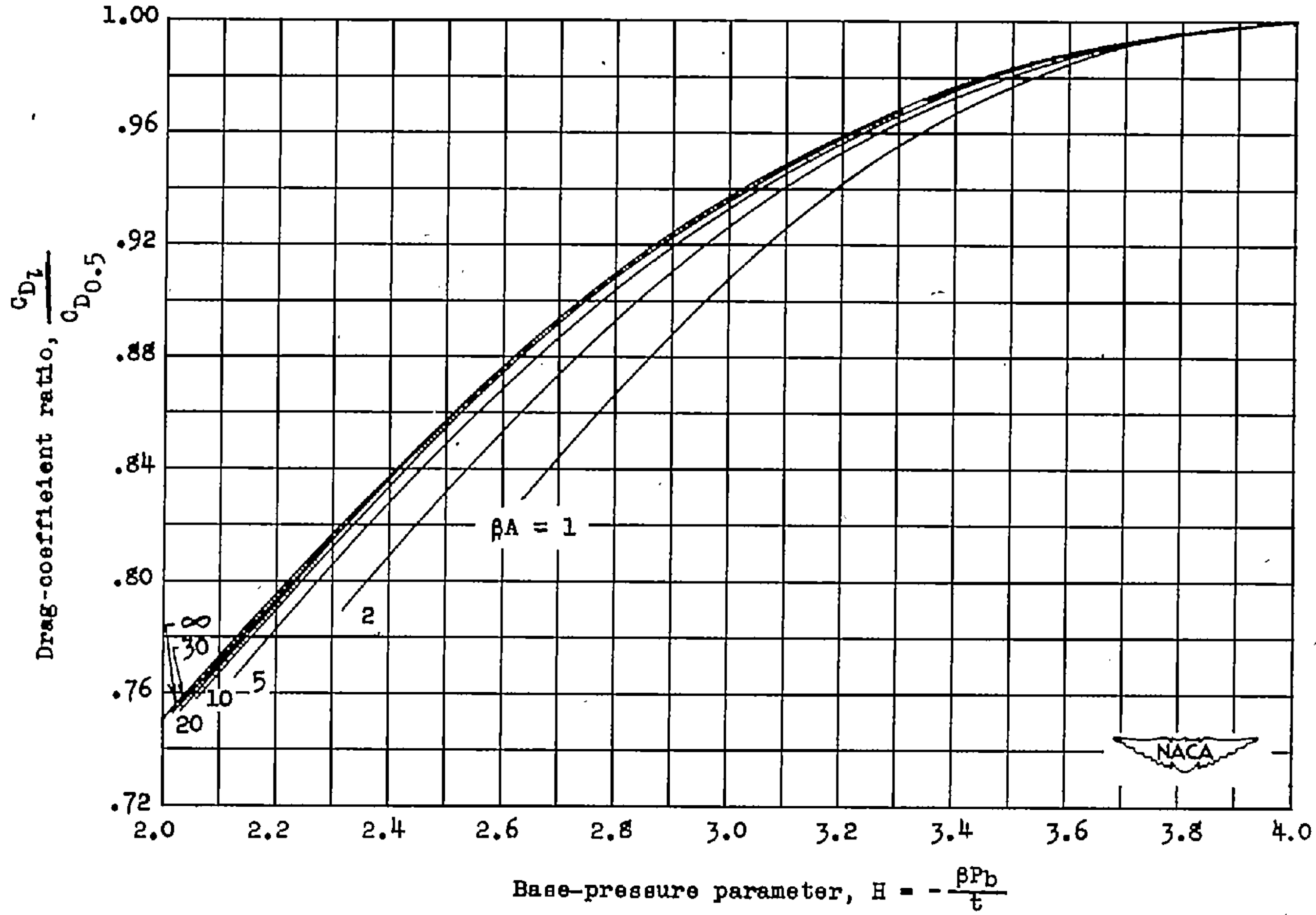
(a) Rectangular wing.

Figure 2.- Position of maximum thickness and trailing-edge thickness for minimum drag as a function of the base-pressure parameter.



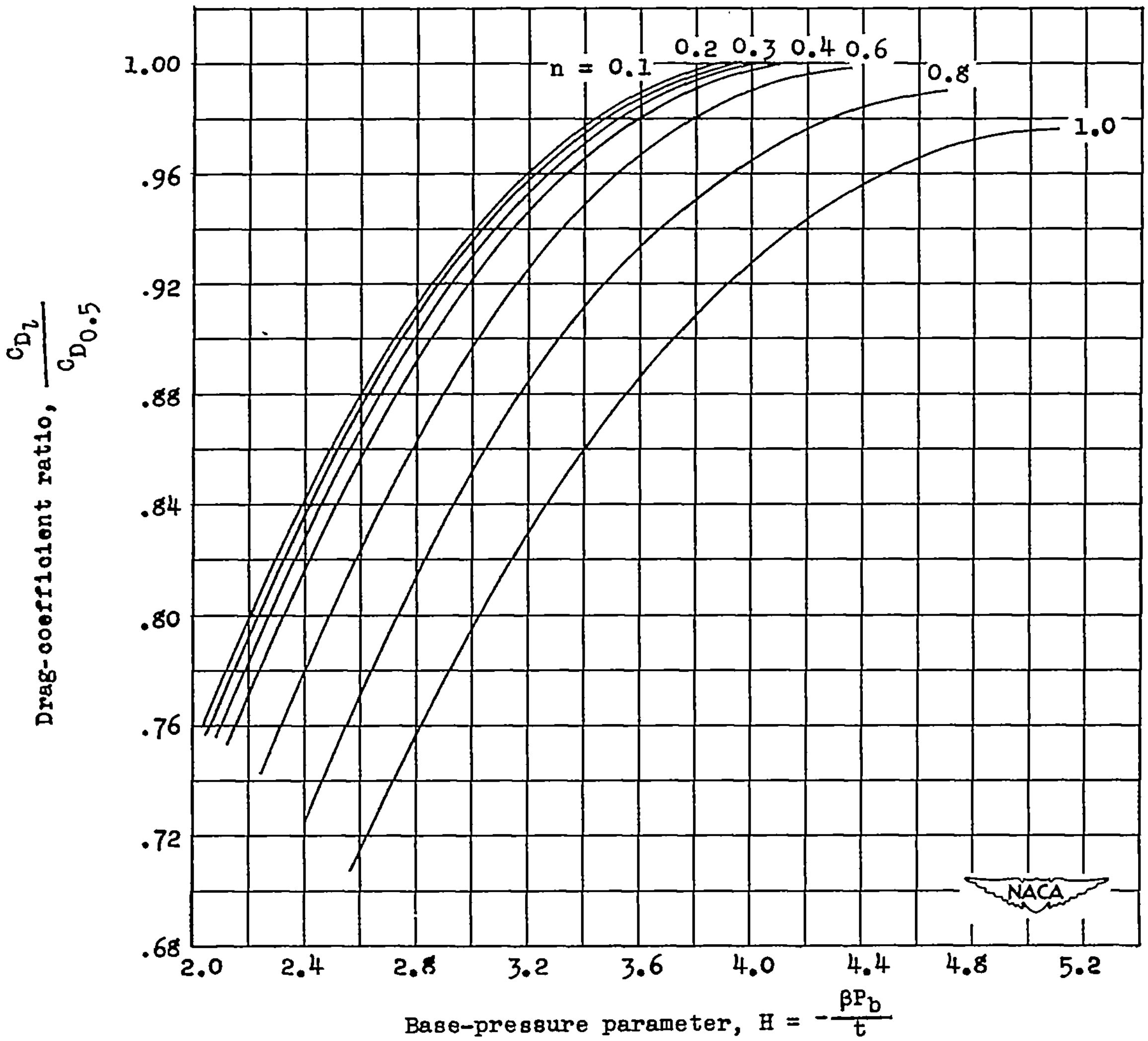
(b) Delta wing.

Figure 2.- Concluded.



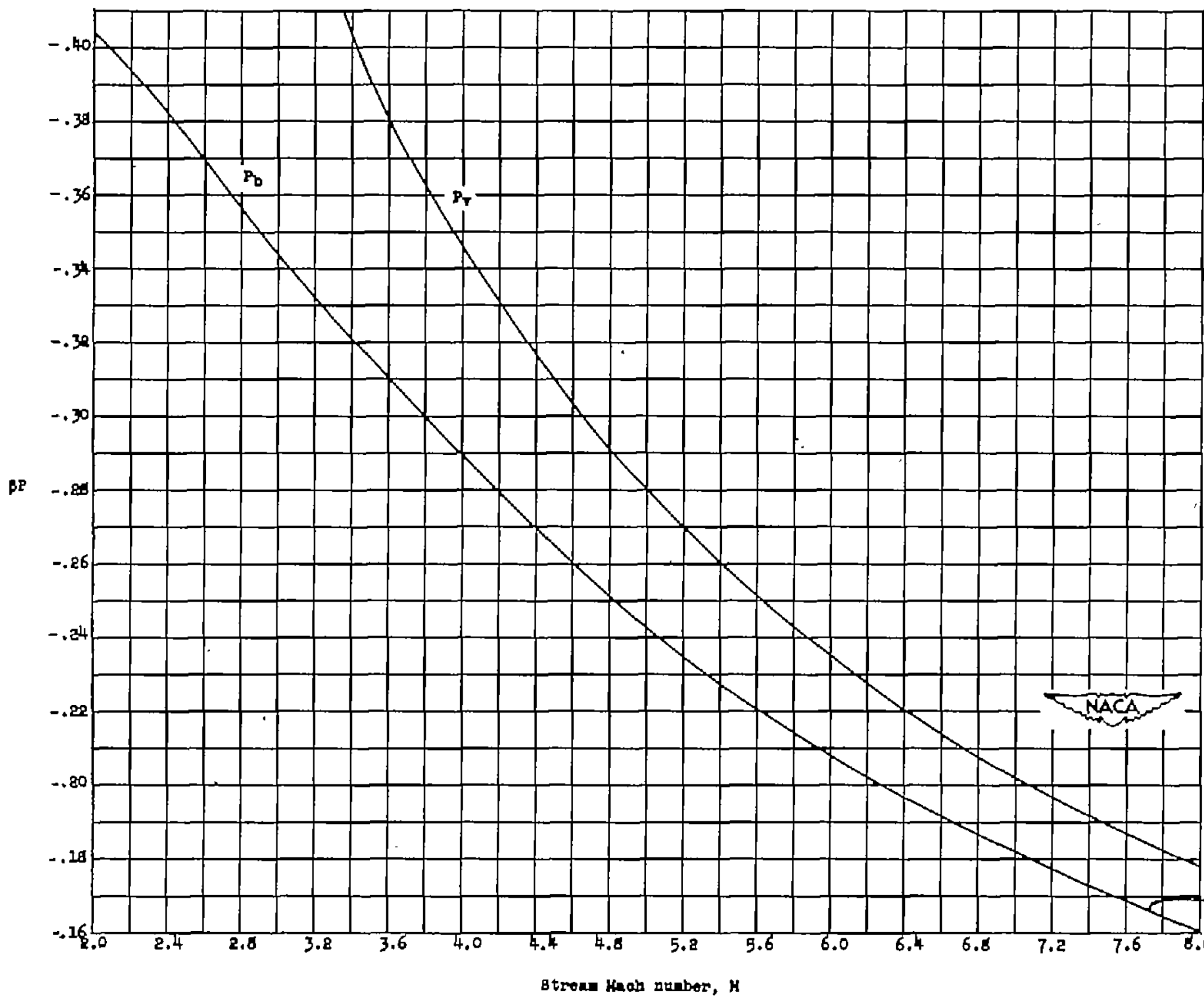
(a) Rectangular wing.

Figure 3.- Ratio of minimum drag coefficient of a wing with a blunt trailing edge to drag coefficient of a wing with a double-wedge section as a function of the base-pressure parameter.



(b) Delta wing.

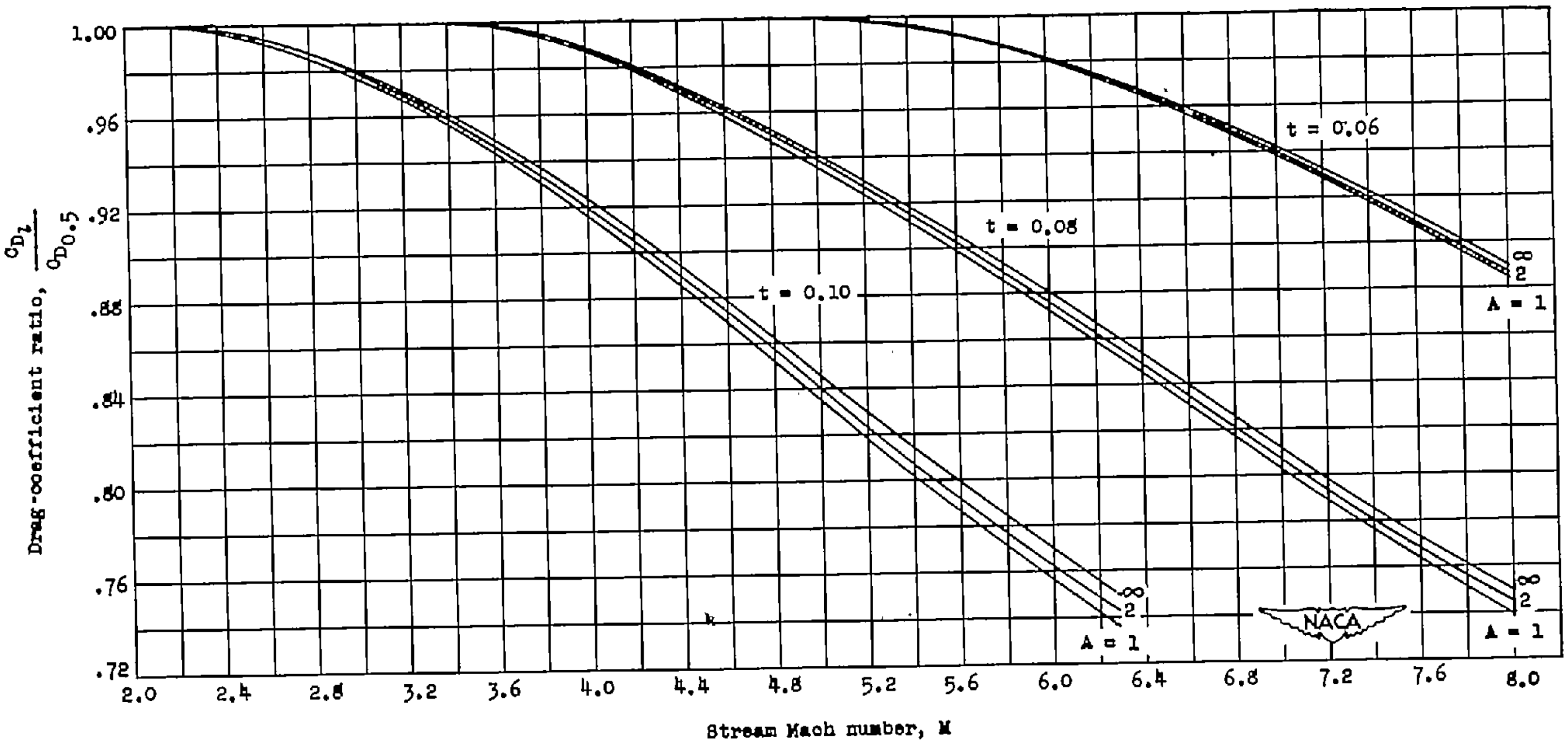
Figure 3.- Concluded.



KLUNKER & HARDER
NACA TN 2623, 1952

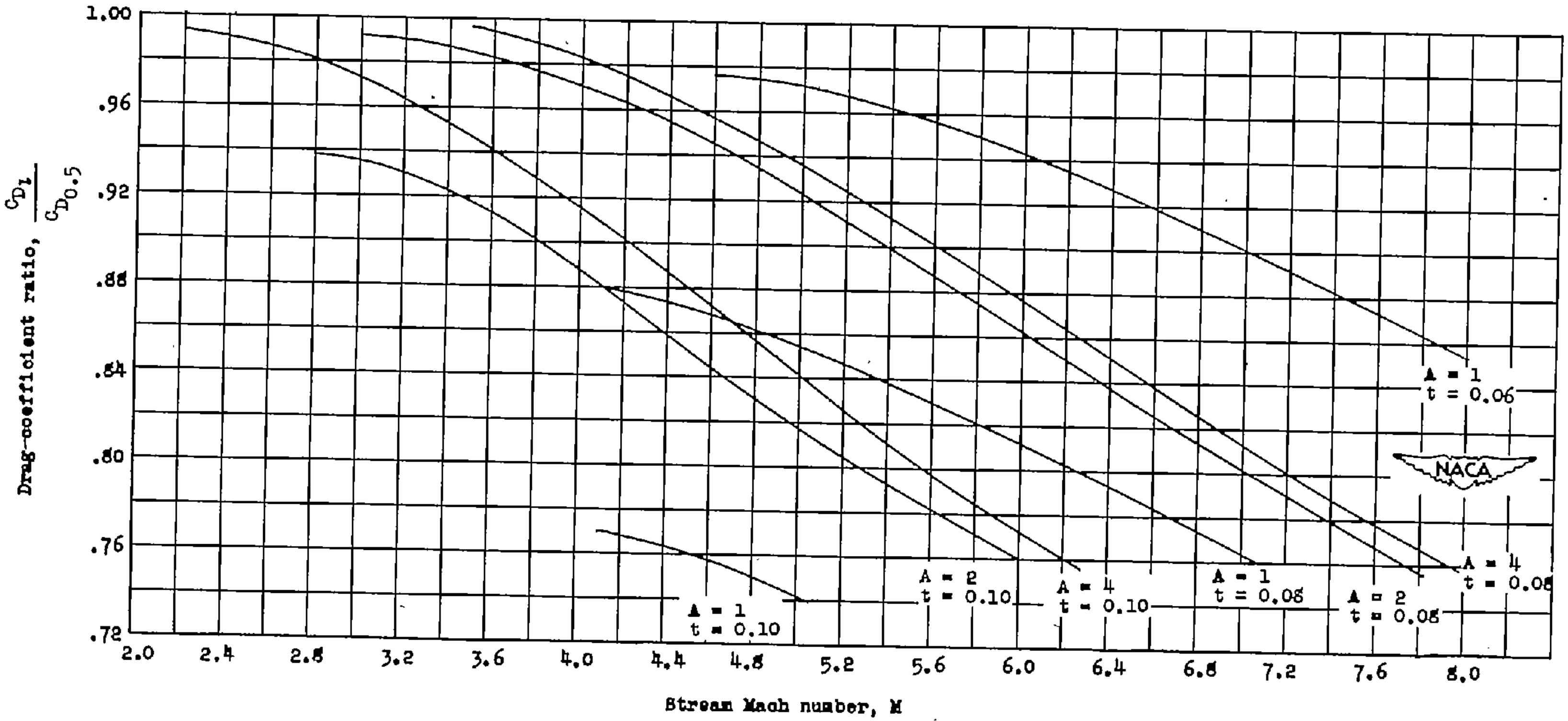
Figure 4.- Vacuum pressure coefficient and base pressure coefficient as functions of stream Mach number.

See p 12, first paragraph.



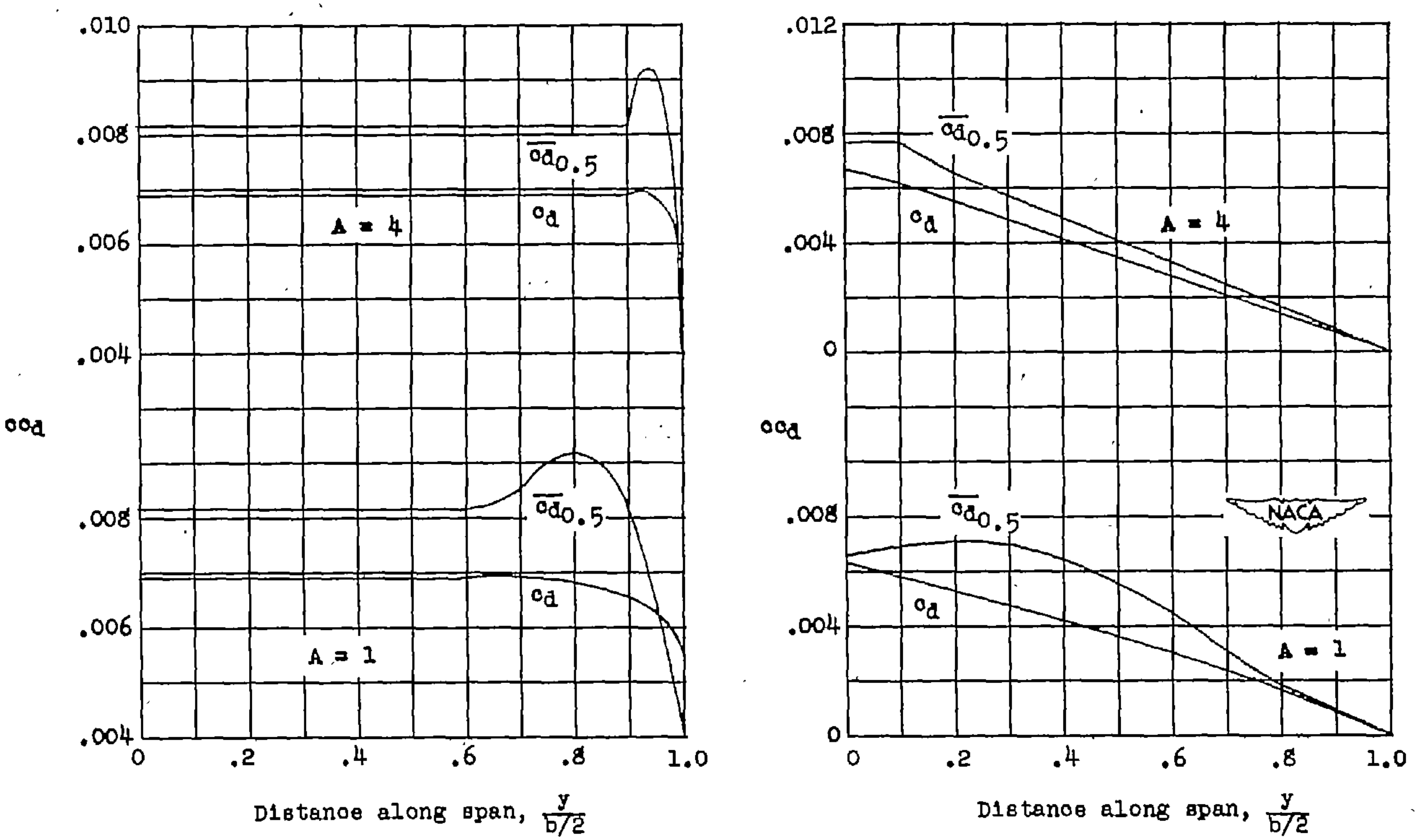
(a) Rectangular wing.

Figure 5.- Ratio of minimum drag coefficient of a wing with a blunt trailing edge to drag coefficient of a wing with a double-wedge section as a function of the stream Mach number for base pressures presented in figure 4.



(b) Delta wing.

Figure 5.- Concluded.



(a) Rectangular wing.

(b) Delta wing.

Figure 6.- Comparison of spanwise distribution of drag for rectangular and delta wings of least drag with rectangular and delta wings having sharp trailing edge and position of maximum thickness at midchord. $M = 5$; $t = 0.10$.